

Heavy Quarkonium and QCD Nonrelativistic Effective Field Theories *

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QCD nonrelativistic effective field theories (NREFT) are the modern and most suitable frame to describe heavy quarkonium properties. In this talk I summarize few relevant concepts and some interesting physical applications of NREFT.

I. INTRODUCTION

Heavy quarkonium systems play a key role in a large range of ongoing or planned experiments, from the search of hybrids to the quarkonium production, from the quark-gluon plasma formation to the next linear collider physics. Being nonrelativistic systems, they enjoy a degree of simplification with respect to the other quark bound systems and thus appear to be the most appropriate laboratory to study the confinement mechanism and the QCD vacuum properties. Being bound systems with a characteristic radius r extending from the short range ($r < 1/\Lambda_{\text{QCD}}$) to the long range ($r > 1/\Lambda_{\text{QCD}}$) they probe the transition region from the perturbative to the nonperturbative regime. For all these reasons, it is relevant to be able to treat these systems inside QCD and in a model independent approach. In particular, to relate the properties of heavy quarkonium directly to the fundamental QCD parameters, like α_s and Λ_{QCD} .

The study of heavy quark-antiquark systems is an old topic, see [1] for some reviews. The spectra show that the gap between the energy levels of such systems is much smaller than the mass of the constituent quarks. Thus, they are nonrelativistic systems and can be described in first approximation by a Schrödinger equation with a potential. The form of the potential may be phenomenologically constrained by the structure of the energy levels which points to something intermediate between a Coulomb and a harmonic oscillator potential. A whole

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zoo of phenomenologically inspired potentials has been used in the past to reproduce/predict the properties of the spectra. The main ingredients of such potentials remain a Coulomb term superimposed with a linear (confining) term [1]. But in spite of their success [2, 3], the limitations of such phenomenological potential models are clear: the theoretical understanding is poor, there is no room for systematic improvement, there is no clear relation between the parameters of the potential models and the fundamental parameters of QCD. In such approaches the confining interaction is imposed by hand. Thus, relativistic corrections to the static potential are added typically in a complete model dependent way and there is no systematic procedure to take into account retardation effects which are typically related to low energy gluons. This last thing becomes particularly relevant in QCD where nonperturbative contributions may appear also as nonpotential effects (carried e.g. by the gluon condensate [4]). This lead in the past to many inconsistencies and contradictory statements about the existence or not of the $q\bar{q}$ potential.

A more rigorous method to obtain the potentials from QCD was developed inside the Wilson loop approach where the potentials are calculated as vacuum expectation values of Wilson loops and (chromo)electric and (chromo)magnetic field insertions inside Wilson loops [1, 5]. Such objects are suitable for a direct lattice evaluation or a calculation inside QCD vacuum models. However, also this approach is missing part of the dynamics that may characterize a particular heavy quarkonium state, like some of the short distance higher order contributions and nonpotential effects.

It appears that in the several approaches to quarkonium physics there is always something out of control. This is related, on one hand to the many physical scales that enter the problem and control several dynamic effects: perturbative (hard scale) and nonperturbative (low scale) effects; potential and nonpotential contributions, local and nonlocal condensates contributions. On the other hand, it is due to the lack of a fully systematic approach endowed with a unambiguous power counting (in some small parameter) that allows to estimate clearly the order of magnitude, and thus the relevance, of the neglected contributions. The effective field theory (EFT) approach satisfactory eliminates such difficulties [6, 7, 9, 10]. Indeed, the existence of a hierarchy of energy scales in quarkonium systems allows the construction of EFT with less and less degrees of freedom. This leads ultimately to a field theory derived quantum mechanical description of these systems. We call pNRQCD (potential Non Relativistic QCD) the corresponding EFT [6, 9, 10]. It is important to stress

that all the EFT that we will introduce here are, by construction, completely equivalent to QCD. The procedure through which such an equivalence is imposed and the integration of the degrees of freedom is done in practice, is called “matching” [6].

To be able to disentangle the scales of the bound state, as the EFT allows us to do, is of key importance in QCD, where we have a confinement region and we would like to be able to ‘factorize’ as much as possible the high energy physics from the low energy physics, dominated by nonperturbative effects. Even inside a pure lattice approach, we have to resort to the EFT approach in order to be able to eliminate the non relevant scales and thus make the heavy quarkonium system fits inside the present capabilities of lattice QCD [8].

Due to space limitations, the present paper is only a guided (and partial) collection of recent results and references in QCD NREFT for heavy quarkonium. The reader is warmly suggested to refer to the quoted papers for all the details and the explanations.

II. EFFECTIVE FIELD THEORIES FOR HEAVY QUARKONIUM

Being nonrelativistic, heavy quarkonium systems are characterized by, at least, three widely separated scales: the mass m of the quark, the (soft) scale associated with the relative momentum $p \sim mv$, $v \ll 1$, and the (ultrasoft) scale associated with the typical kinetic energy $E \sim mv^2$. Also the inverse of the typical size of the system r^{-1} is of order mv . Here v is the velocity of the quark in the bound system and what matters for the following is only that v is a small number. The power counting of the EFT will be established in powers of v . This point requires special care in the case of charmonium where it is not clear if v remains a sufficiently small number. Moreover, in dependence of the specific system, the scale of the nonperturbative physics Λ_{QCD} , may turn out to be close to some of the above dynamical scales. The physical picture, which then arises, may be quite different from the perturbative situation. What remains true for all heavy quarkonia is that $m \gg \Lambda_{\text{QCD}}$ and thus at least the mass scale can be treated perturbatively, i.e. integrated out from QCD order by order in the coupling constant α_s . The resulting EFT is called NRQCD (Non Relativistic QCD) [7]. The Lagrangian of NRQCD can be organized in powers of $1/m$, thus making explicit the non-relativistic nature of the physical systems. In order for an effective field theory to be useful, a power counting is needed. The power counting of NRQCD (organized in powers of v and α_s) follows from arguments valid in the perturbative regime

(which should correspond strictly speaking to $\Lambda_{\text{QCD}} \lesssim mv^2$)[6]. Moreover, being still two scales (the momentum and energy scales) dynamical, the matrix elements do not have a unique power counting beyond leading order [9]. NRQCD allows us to calculate on the lattice systems like bottomonium [8]. The new and very successful predictions of NRQCD on inclusive quarkonium decays and on quarkonium production are well known [7].

III. SMALL RADIUS SYSTEMS AND PNRQCD (FOR $mv \gg \Lambda_{\text{QCD}}$)

In NRQCD still the dominant role of the potential as well as the quantum mechanical nature of the problem are not yet maximally exploited. A higher degree of simplification may still be achieved. In other words, we want to build another effective theory for the low energy region of the non-relativistic bound-state, i.e. we want an EFT where only the ultrasoft degrees of freedom remain dynamical, while the unwanted degrees of freedom are integrated out. To this aim we integrate out the scale of the momentum transfer $\sim mv$ which is supposed to be the next relevant scale. Then, two different situations may exist. In the first one, $mv > \Lambda_{\text{QCD}}$ and thus the matching from NRQCD to pNRQCD may be performed in perturbation theory, expanding in α_s . This is the situation that I will discuss in this paragraph. In the second situation, $mv \lesssim \Lambda_{\text{QCD}}$, the matching has to be nonperturbative, i.e. no expansion in α_s is allowed. Recalling that $r^{-1} \sim mv$, these two situations correspond to systems with inverse typical radius smaller or bigger than Λ_{QCD} , or systems respectively dominated by the short range or long range (with respect to the confinement radius) physics. Although no direct measurements of the typical radius is possible, from all the information we have at hand we can say that charmonium belongs to the second case and we will discuss it together with the nonperturbative matching to pNRQCD in Sec.4.

Now, we briefly describe the case in which $mv > \Lambda_{\text{QCD}}$. At the scale of the matching μ' ($mv \gg \mu' \gg mv^2, \Lambda_{\text{QCD}}$) we have still quarks and gluons. The effective degrees of freedom are: $Q\bar{Q}$ states (that can be decomposed into a singlet and an octet wave function under color transformations) with energy of order of the next relevant scale, $O(\Lambda_{\text{QCD}}, mv^2)$ and momentum \mathbf{p} of order $O(mv)$, plus ultrasoft gluons $A_\mu(\mathbf{R}, t)$ with energy and momentum of order $O(\Lambda_{\text{QCD}}, mv^2)$. All the gluon fields are multipole expanded (i.e. in r). The Lagrangian is then an expansion in the small quantities p/m , $1/rm$ and in $O(\Lambda_{\text{QCD}}, mv^2) \times r$.

The EFT we obtain [6, 9, 10] produces a zero order equation and correction interactions

terms of the type [6, 10]

$$\left. \begin{aligned} \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_0(r) \right) \Phi(\mathbf{r}) &= 0 && + \text{corrections to the potential} \\ + \text{interaction with other low-energy degrees of freedom} &&& \end{aligned} \right\} \text{pNRQCD}$$

where $V_0(r) = -C_f\alpha_s/r$ in the perturbative tree level case for the singlet and $\Phi(\mathbf{r})$ is the \bar{Q} - Q (singlet or octet) wave-function.

The equivalence of pNRQCD to NRQCD, and hence to QCD, is enforced by requiring the Green functions of both effective theories to be equal (matching). In practice, appropriate off shell amplitudes are compared in NRQCD and in pNRQCD, order by order in the expansion in $1/m$, α_s and in the multipole expansion. The difference is encoded in potential-like matching coefficients that depend non-analytically on the scale that has been integrated out (in this case \mathbf{r}).

At the leading order (LO) in the multipole expansion, the equations of motion of the singlet field is the Schrödinger equation. Therefore pNRQCD has made explicit the dominant role of the potential and the quantum mechanical nature of the bound state. In particular both the kinetic energy and the potential count as mv^2 in the v power counting. The actual bound state calculation turns out to be very similar to a standard quantum mechanical calculation, the only difference being that the wave function field couples to US gluons in a field theoretical fashion. From the solution of the Schrödinger equation we obtain the leading order propagators for the singlet and the octet state, while the vertexes come from the interaction terms at the NLO (next-to-leading order) in the multipole expansion. In fact the pNRQCD Lagrangian contains retardation (or nonpotential) effects that start at the NLO in the multipole expansion. Thus, pNRQCD has explicit potential terms and thus it embraces a description of heavy quarkonium in terms of potentials. However, it has also explicit dynamical ultrasoft gluons and thus it describes nonpotential (retardation) effects. As we see, such an effective theory is able to provide a solution to the problems mentioned in the introduction.

The power counting is unambiguous. Calculations can be performed systematically in the v expansion and can be improved at the desired order. Perturbative (high energy) and nonperturbative (low energy) contributions are disentangled. Renormalization group improvement may be performed in the effective theory[11].

This allows us to systematically parameterize the nonperturbative contributions that we

are not able to evaluate directly. There are two main situations. If $mv^2 \leq \Lambda_{\text{QCD}}$ the system is described up to order α_s^4 by a potential entirely accessible to perturbative QCD. Nonpotential effects start at order $\alpha_s^5 \ln \mu'$ and have been calculated in [12]. We call Coulombic this kind of systems. Nonperturbative effects are of non potential type and can be encoded into local (à la Voloshin-Leutwyler) or nonlocal condensates. In the second case, when $mv \gg \Lambda_{\text{QCD}} \gg mv^2$, nonperturbative contributions to the potential arise when integrating out the scale Λ_{QCD} [10]. We call quasi-Coulombic the systems where the nonperturbative piece of the potential can be considered small with respect to the Coulombic one. Some levels of toponium, the lowest level of $b\bar{b}$ may be considered Coulombic systems, while the J/ψ , the η_c , the lowest level of B_c and part of the bottomonium excited levels maybe considered as quasi-Coulombic. Detailed calculation of the properties of such systems in this frame may be found in [13, 14, 15]. In particular in [13] an accurate determination of the mass of the b is also obtained.

As it is typical in an effective theory, only the actual calculation may confirm if the initial assumption about the physical system was appropriate.

IV. CHARMONIUM AND THE NONPERTURBATIVE MATCHING TO PNRQCD

With the exception of the lowest state, which maybe a quasi-Coulombic system, the main part of the excited levels of charmonium probe the confinement region $mv \lesssim \Lambda_{\text{QCD}}$. Then, pNRQCD should be obtained via a nonperturbative matching, i.e. without expansions in α_s . This have been proved to be equivalent to compute the heavy quarkonium potential at order $1/m^2$ [16]. More precisely, a pure potential picture emerges at the leading order in the ultrasoft expansion under the condition that all the gluonic excitations (hybrids) have a gap of order Λ_{QCD} [16]. Higher order effects in the $1/m$ expansion as well as extra ultrasoft degrees of freedom such as hybrids or pions can be systematically included and may eventually affect the leading potential picture. Thus we recover the quark model from pNRQCD[16]. The final result for the potentials (static and relativistic corrections) appear factorized in a part containing the high energy dynamics (and calculable in perturbation theory) which is inherited from NRQCD, and a part containing the low energy dynamics given in terms of Wilson loops and chromo-electric and chromo-magnetic insertions in the Wilson loop [1, 6, 16]. Such low energy contributions can be simply calculated on the lattice

or evaluated in QCD vacuum models. Also in this case the power counting supply us with a valuable and systematic way of estimating the size of the neglected terms. Moreover, since the power counting of pNRQCD may be different from the power counting of NRQCD, we expect that we may eventually explain in this way the difficulties that NRQCD is facing in explaining the polarization of the prompt J/ψ data [18]. New and quite interesting predictions have been obtained in this frame also for charmonium P wave inclusive decays [17] (see also [19]) and on the behaviour of the heavy (and thus also charmed) hybrids potential for small r [10].

V. CONCLUSIONS

An effective theory of QCD which describes heavy quarkonium has been constructed systematically and within a controlled expansion. Such a theory disentangles the scales of the bound state and has a definite power counting. All known perturbative and nonperturbative, potential and nonpotential regimes are present and separately factorized in the theory. In this way the properties of heavy quarkonium and in particular of charmonium, which is the subject of the present conference, are related to the fundamental parameters of QCD. Such an effective theory is thus the appropriate frame to perform calculation of heavy quarkonium properties. In this paper we presented just a guided recollection of recent references where new results on heavy quarkonium spectra, decays, production and heavy quarkonium hybrids potentials can be found inside the frame of pNRQCD.

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